Introduction

Numeracy is vital to all aspects of everyday life. All pupils should leave school both literate and numerate. The Scottish Government has released a new curriculum in the form of 'A Curriculum for Excellence'. Within this new curriculum all teachers have the responsibility to teach both literacy and numeracy.

With this in mind a numeracy committee was formed with the remit to standardise the teaching of numeracy throughout the school and at home. This booklet contains teaching methods which should be used throughout the school to prevent confusion and to build a firm understanding of each numeracy topic.

1. Estimation and Rounding

Level 2 I can use my knowledge of rounding to routinely estimate the answer to a problem, then after calculating, decide if my answer is reasonable, sharing my solution with others **MNU 2-01a**

Level 3 I can round a number using an appropriate degree of accuracy, having taken into account the context of the problem. **MNU 3-01a**

Level 4 Having investigated the practical impact of inaccuracy and error, I can use my knowledge of tolerance when choosing the required degree of accuracy to make real life calculations. **MNU 4-01a**

Rounding

General rule:

> *"Half way or more - round up"*

Questions:

What number is 'half way' between 0 and 10?
What number is 'half way' between 0 and 100?
What number is 'half way' between 0 and 1000? Etc…

Examples

Round 34 to the nearest 10 → 30
Round 258 to the nearest 100 → 300
Round 3850 to the nearest 1000 → 4000

Estimation

Estimation can and should be used in everyday contexts. Examples of questions could include:

'How much do you think shopping will cost?'
'Roughly how many Euros do you think I would get for £200?'
'How many people does the National Stadium hold?'

Tolerance

Tolerance is regarded as a more technical term and would only be used in certain subject areas. It is also relevant in an industrial context.

Example:

Length = 20mm ± 0.5mm

This statement would indicate that the length could therefore lie between 19.5mm and 20.5mm.

2. Number and number processes

Level 2

I have extended the range of whole numbers I can work with and having explored how decimal fractions are constructed, can explain the link between a digit, its place and its value. **MNU 2-02a**

Having determined which calculations are needed, I can solve problems involving whole numbers using a range of methods, sharing my approaches and solutions with others. **MNU 2-03a**

I have explored the contexts in which problems involving decimal fractions occur and can solve related problems using a variety of methods. **MNU 2-03b**

I can show my understanding of how the number line extends to include numbers less than zero and have investigated how these numbers occur and are used. **MNU 2-04a**

Level3

I can use a variety of methods to solve number problems in familiar contexts, clearly communicating my processes and solutions. **MNU 3-03a**

I can continue to recall number facts quickly and use them accurately when making calculations. **MNU 3-03b**

I can use my understanding of numbers less than zero to solve simple problems in context. **MNU 3-04a**

Level 4

Having recognised similarities between new problems and problems I have solved before, I can carry out the necessary calculations to solve problems set in unfamiliar contexts. **MNU 4-03a**

The link between fractions and decimals

Pupils should be aware that both fractions and decimal fractions represent part of a whole one. They should also understand the link between fractions and decimals. A straight forward illustration is to show place value and the equivalent fraction i.e.

Units	.	Tenths	Hundredths	Thousandths	
0	.	1			$= \frac{1}{10}$
0	.	0	1		$= \frac{1}{100}$
0	.	0	0	1	$= \frac{1}{1000}$

We can write any fraction in decimal form as follows:

Example: Write $\frac{3}{8}$ as a decimal fraction

Calculator method

$$\frac{3}{8}$$
$$= 3 \div 8$$
$$= 0.375$$

Non Calculator method

i. $\frac{3}{8} = \frac{\quad}{10} = \frac{\quad}{100} = \frac{375}{1000} = 0.375$

ii. $8\overline{)3.000}$ $\quad 0.375$

Subtraction by decomposition

Example 1

```
  2234
- 1598
  0636
```

Example 2

```
  3000
- 2598
  0402
```

Numbers less than zero

Numbers less than zero can be discussed in context i.e. temperature, sea level and bank accounts. Within Mathematics pupils are also encouraged to use a number line to help with calculations. This number line is just as on a thermometer and is also used within the topic co-ordinates.

To add a number to either a positive or negative number move right along the number line and to subtract move left along the number line.

Order of Operations

Operations should be carried out in the following order:

B – brackets
O – operations
D – division
M – multiplication
A – addition
S - subtraction

Example:

Incorrect	Correct
$6+2\times4=32$	$6+2\times4=14$

(the multiplication is carried out before the addition)

Mental Strategies

Mental strategies are essential for everyday life. Pupils are taught various strategies to add and subtract in primary school the most common being as follows:

e.g. 86 + 19 Pupils would be encouraged to first add 20 then take one away. Therefore 86 + 20 = 106 – 1 = 105

This can also be used for subtraction. E.g. 94 – 18 Pupils would be encouraged to take away 20 then add 2 back on. Therefore 94 – 20 = 74 + 2 = 76

It was also noted that pupils tend to be good at calculations when they are placed in the context of money!

Finally, mental strategies rely on a good knowledge and recall of times tables. These should be practiced regularly.

3. Fractions, Decimals and Percentages

Level 2

I have investigated the everyday contexts in which simple fractions, percentages or decimal fractions are used and can carry out the necessary calculations to solve related problems. **MNU 2-07a**

I can show the equivalent forms of simple fractions, decimal fractions and percentages and can choose my preferred form when solving a problem, explaining my choice of method. **MNU 2-07b**

Level 3

I can solve problems by carrying out calculations with a wide range of fractions, decimal fractions and percentages, using my answers to make comparisons and informed choices for real life situations. **MNU 3-07a**

I can show how quantities that are related can be increased or decreased proportionally and apply this to solve problems in everyday contexts. **MNU 3-08a**

Level 4

I can choose the most appropriate form of fractions, decimal fractions and percentages to use when making calculations mentally, in written form or using technology, then use my solutions to make comparisons, decisions and choices. **MNU 4-07a**

Using proportion, I can calculate the change in one quantity caused by a change in a related quantity and solve real life problems. **MNU 4-08a**

Converting fractions to Percentages

All pupils should know how to convert % to fraction to decimal. The main idea behind linking fractions, decimal and % is that of:

% means 'out of 100'

Once this is established fractions can be easily written, and simplified, and then knowledge of place value can be used to write hundredths directly to decimal form.

Examples:

1. $37\% = \frac{37}{100} = 0.37$

2. $9\% = \frac{9}{100} = 0.09$

When calculating say 50% or 25% pupils should be aware of the fraction equivalent as this provides an easier calculation which can be carried out mentally. Some fractions don't produce 'nice' decimals e.g. $33\frac{1}{3}\% = \frac{1}{3}$, is better left in fraction form to provide a more accurate answer as once the decimal is rounded accuracy is lost.

Commonly used percentage equivalences should be remembered by all:

%	Fraction	Decimal
50%	$\frac{50}{100} = \frac{1}{2}$	0.50
25%	$\frac{25}{100} = \frac{1}{4}$	0.25
75%	$\frac{75}{100} = \frac{3}{4}$	0.75
20%	$\frac{20}{100} = \frac{1}{5}$	0.20
10%	$\frac{10}{100} = \frac{1}{10}$	0.10
1%	$\frac{1}{100}$	0.01
33 1/3 %	$\frac{33\frac{1}{3}}{100} = \frac{1}{3}$	0.333333 (not used)
66 2/3 %	$\frac{66\frac{2}{3}}{100} = \frac{2}{3}$	0.666667 (not used)

Decimal equivalency can also be discussed using place value as a one percent is one hundredth and so on.

Real life situations include banks, insurance, sales, concentration of chemicals etc.

Finding a Percentage of an amount

Method 1: Without a calculator

The above table of equivalent fractions should be used to calculate any percentage. For example
Calculate 63% of £250

- 50% of £250 = £250 / 2 = £125
- 10% of £250 = £250 / 10= £ 25
- 1% of £250 = £250 / 100=£ 2.50

Therefore 63% = £125 + £25 + £2.50 + £2.50 + £2.50
= £157.50

Method 2: With a calculator

The statement "% means out of 100" is used to perform calculations with a calculator.

Example

Calculate 58% of £24700.

$= \frac{58}{100} \times 24700$

$= 58 \div 100 \times 24700$

$= 14326$

One Amount as a Percentage of Another

All pupils should be encouraged to work out a percentage from their **test scores** as this would enable comparisons of attainment across subject areas.

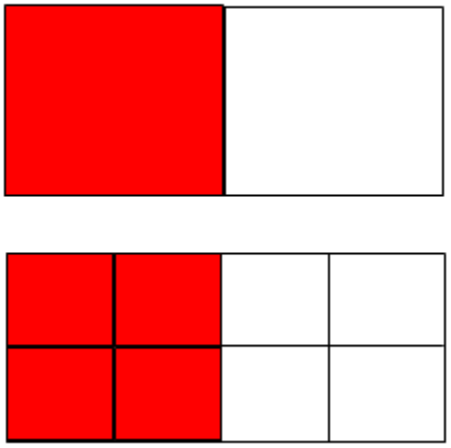
Example

Joe gained 35 out of 40 in his Maths test. Write his mark as a percentage.

$$\frac{35}{40} = 35 \div 40 \times 100 = 87.5\%$$

Simplest form and Equivalent fractions

In the first instance equivalent fractions can be illustrated pictorially as follows:



$\frac{1}{2}$

$\frac{4}{8}$

$$\frac{1}{2} = \frac{4}{8}$$

Once an understanding is established, mathematical approaches can be taught. Pupils are taught that whatever calculation is performed on the numerator (top number) should be performed on the denominator (bottom number) and vice versa e.g.

1. $\frac{2}{3}=\frac{2\times4}{3\times4}=\frac{8}{12}$ 2. $\frac{4}{5}=\frac{4\times6}{5\times6}=\frac{24}{30}$

Mixed Numbers to Improper Fractions

"Multiply bottom add top" e.g. 1. $2\frac{1}{4}=\frac{9}{4}$ 2. $5\frac{2}{3}=\frac{17}{3}$

$(4\times2+1=9)$ $(3\times5+2=17)$

Improper Fractions to Mixed Numbers

"How many times does the bottom go into the top and how many are left?"

e.g. 1. $\frac{13}{2}=6\frac{1}{2}$ 2. $\frac{30}{4}=7\frac{2}{4}=7\frac{1}{2}$

Finding a fraction of an amount

To find a fraction of an amount simply divide by the 'bottom number' and times by the 'top'.

Example $\frac{2}{3}$ of 24

24 ÷ 3 = 8

8 × 2 = 16

Adding and Subtracting Fractions

Adding and subtracting fractions should be taught using a common denominator, although the 'kiss smile' method could be taught as an emergency back-up method! Examples:

1. Using a common denominator

$$\frac{2}{3}+\frac{1}{5}=$$
$$\frac{10}{15}+\frac{3}{15}=$$
$$\frac{13}{15}$$

2. 'Kiss Smile'

$$\frac{2}{3}\times\frac{1}{5}=$$
$$\frac{(2*5)+(1*3)}{(3*5)}=$$
$$\frac{13}{15}$$

Multiply fractions

Multiplying fractions is easiest taught by simply multiplying the numerators (top number) together and then multiplying the denominators (bottom numbers) together. Some pupils may have been taught how to 'cancel down' but this is only used for credit equivalent classes. It is important that fractions are simplified at the end of the calculation.

Examples

1. $\frac{4}{5}\times\frac{2}{3}=\frac{4\times2}{5\times3}=\frac{8}{15}$

2. $\frac{2}{9}\times\frac{3}{4}=\frac{2\times3}{9\times4}=\frac{6}{36}=\frac{1}{6}$

Dividing Fractions

To divide by a fraction simply turn the dividing fraction up-side-down and multiply.

Examples

1. $\frac{4}{9} \div \frac{2}{3} = \frac{4}{9} \times \frac{3}{2} = \frac{12}{18} = \frac{2}{3}$

2. $\frac{6}{7} \div \frac{3}{5} = \frac{6}{7} \times \frac{5}{3} = \frac{30}{21} = 1\frac{9}{21} = 1\frac{3}{7}$

Decimals

Understanding Place Value

Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tenths	Units	.	Tenths	Hundredths	Thousandths
0	0	0	0	0	0	0	.	0	0	0

Reading Numbers

- Grouping numbers in 3's allows us to read millions and thousands easily.
- Numbers after the point should be read as single digits, not read as a number!

Example

7625317.52 reads: "Seven million, six hundred and twenty five thousand, three hundred and seventeen point five two"

The Four Operations ($+ - \times \div$)

When working with decimals the decimal point should be placed carefully before performing the calculation. Working in columns will ensure correct place value.

Examples

+

$$\begin{array}{r} 45.02 \\ +\ \ 3.687 \\ \hline \end{array}$$

−

$$\begin{array}{r} 38.00 \\ -\ \ 5.62 \\ \hline \end{array}$$

×

$$\begin{array}{r} 45.2 \\ \times\ \ 6 \\ \hline \end{array}$$

÷

$$5\overline{)55.865}$$

Remember:

- Line up the decimals points!

Note: Pupils tend to understand decimals with ease when placed in the context of money!